

Josephson effect in Graphene SNS Junction with a Single Localized Defect

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Imperfections change essentially the electronic transport properties of graphene. Motivated by a recent experiment reporting on the possible application of graphene as junctions, we study transport properties in graphene-based junctions with single localized defect. We solve the Dirac-Bogoliubov-de-Gennes equation with a single localized defect superconductor-normal(graphene)-superconductor (SNS) junction. We consider the properties of tunneling conductance and Josephson current through an undoped strip of graphene with heavily doped *s*-wave superconducting electrodes in the limit $l_{def} \ll L \ll \xi$. We find that spectrum of Andreev bound states are modified in the presence of single localized defect in the bulk and the minimum tunneling conductance remains the same. The Josephson junction exhibits sign oscillations.

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I. INTRODUCTION

Recent exciting developments in transport experiments on graphene has stimulated theoretical studies of superconductivity phenomena in this material, which has been recently fabricated [1, 2]. A number of unusual features [3] of superconducting state have been predicted [4, 5] which are closely related to the Dirac-like spectrum of normal state excitation [6–9]. In particular, the unconventional normal electron dispersion [10] has been shown to result in a nontrivial modification of Andreev reflection and Andreev bound states in Josephson junctions [11] with superconducting graphene electrodes [12, 13].

Other interesting consequences of the existence of Dirac-like quasiparticles can be understood by studying superconductivity [14] in graphene [15–19]. It has been suggested that superconductivity can be induced in graphene layer in the presence of a superconducting electrode near it via proximity effect [20–22].

In this work, we study Josephson effect and find bound state in graphene [23] for tunneling SNS junction with the presence of a single localized defect [24]. In this study, we shall concentrate on SNS junction with normal region thickness $L \ll \xi$, where ξ is the superconducting coherence length, and width W which has an applied gate voltage U across the normal region [25, 26]. In the frame of the limit $l_{def} \ll L \ll \xi$ considered by Kulik we investigate tunneling conductance in SNS junctions with presence a single localized defect and find that Andreev levels are modified, the minimum tunneling conductance remains the same [27–29].

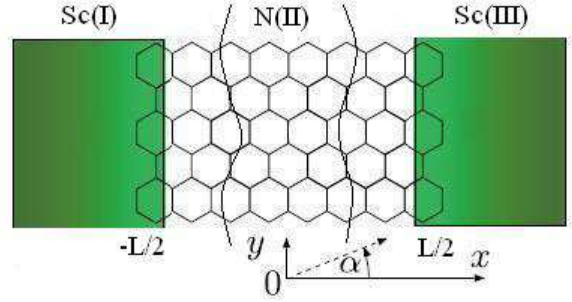


FIG. 1: A graphene undoped ribbon is contacted by two superconducting leads. The charge carriers tunnel from one lead to another via multiple tunneling states formed in the graphene strip. A defect placed inside the strip.

II. JOSEPHSON EFFECT IN SUPERCONDUCTOR/NORMAL(GRAPHENE)/SUPERCONDUCTOR JUNCTIONS WITH A SINGLE LOCALIZED DEFECT

We consider a SNS junction with a single localized defect which is involved in a graphene sheet of width W lying in the $x-y$ plane extends from $x = -L/2$ to $x = L/2$ while the superconducting region occupies $|x| > L/2$ (see Fig. 1). The SNS junctions can then be described by the Dirac-Bogoliubov-de-Gennes (DBdG) equations [30],

$$\begin{pmatrix} H_s - E_F + U & \Delta \\ \Delta^* & E_F - U - H_s \end{pmatrix} \psi_s = \epsilon \psi_s$$

Here, $\psi_s = (\psi_{As}, \psi_{Bs}, \psi_{A\bar{s}}^*, -\psi_{B\bar{s}}^*)$, $\psi = (u_1, u_2, v_1, v_2)$ are the 4 component wave functions for the electron and hole spinors, the index s denote K or K' for electrons or holes near K and K' points, \bar{s} takes values $K(K')$ for $s = K(K')$, E_F denotes the Fermi energy, A and B denote the two inequivalent sites in the hexagonal lattice

of graphene, and the Hamiltonian H_s is given by

$$H_s = -i\hbar v_F [\sigma_x \partial_x + \text{sgn}(s) \sigma_y \partial_y] \quad (1)$$

In Eq. 1, v_F denotes the Fermi velocity of the quasi-particles in graphene and $\text{sgn}(s)$ takes values \pm for $s = K(K')$. The 2×2 Pauli matrices σ_i act on the sublattice index. The excitation energy $\epsilon > 0$ is measured relative to the Fermi level (set at zero). The electrostatic potential U and pair potential Δ have step function profiles, as in the case of a semiconductor two-dimensional electron gas [31–33]: $\Delta(\mathbf{r}) \rightarrow \Delta_0 \exp^{\mp i\phi/2}, -U$ for $x \rightarrow \pm\infty$. We assume non-interacting electrons in the normal region, therefore, $\Delta(\mathbf{r}) \equiv 0$, $U = 0$ for $|x| < L/2$. The reduction of the order parameter $\Delta(x)$ in the superconducting region on approaching the SN interface is neglected; i.e., we approximate parameter $\Delta(x)$ as we have done it above. As discussed by Likharev [34], this approximation is justified if the weak link has length and width much smaller than ξ . There is no lattice mismatch at the NS interface, so the honeycomb lattice of graphene is unperturbed at the boundary, the interface is smooth and impurity free. Zero magnetic field is assumed.

Solving the DBdG equations, we gain the wave-functions in the superconducting and the normal regions. In region $I(III)$, for the DBdG quasiparticles moving along the $\pm x$ direction with a transverse momentum $k_y = q$ and energy ϵ , the wave-functions are given by

$$\Psi^+ = \exp(iqy + ik_s x + \kappa m x) \begin{pmatrix} \exp(-im\beta) \\ \exp(i\gamma - im\beta) \\ \exp(-im\phi/2) \\ \exp(i\gamma - im\phi/2) \end{pmatrix}$$

$$\Psi^- = \exp(iqy - ik_s x + \kappa m x) \begin{pmatrix} \exp(im\beta) \\ \exp(-i\gamma + im\beta) \\ \exp(-im\phi/2) \\ \exp(-i\gamma - im\phi/2) \end{pmatrix}$$

The parameters β , γ , k_0 , κ are defined by $\beta = \arccos(\epsilon/\Delta_0)$, $\gamma = \arcsin[\hbar v_F q / (U_0 + E_F)]$, $k_s = \sqrt{(U_0 + E_F)^2 / (\hbar v_F)^2 - q^2}$, $\kappa = (U_0 + E_F)\Delta_0 \sin(\beta) / (\hbar^2 v_F^2 k_s)$ and $m = \pm$ denotes region $I(III)$, $m = +$ for I and $m = -$ for III correspondently. Further we assumed that the Fermi wave length λ'_F in the superconducting region much smaller than the wave length λ_F in the normal region and $U_0 \gg E_F, \epsilon$. Since $|q| \leq E_F / \hbar v_F$, this regime of a heavily doped superconductor corresponds to the limits $\gamma \rightarrow 0$, $k_s \rightarrow U_0 / \hbar v_F$, $\kappa \rightarrow (\Delta_0 / \hbar v_F) \sin(\beta)$.

Now we analyze the spectral properties of a graphene ring. Formally we find solutions for graphene ring and then extending ring in the scale, match external boarder of ring with superconducting regions of junction and fix internal part implying which as defect. For that we divide the region II into three areas: a, b, c (see Fig. 2). We solve the DBdG equations for the area b , while area c is the area where placed defect and area a would

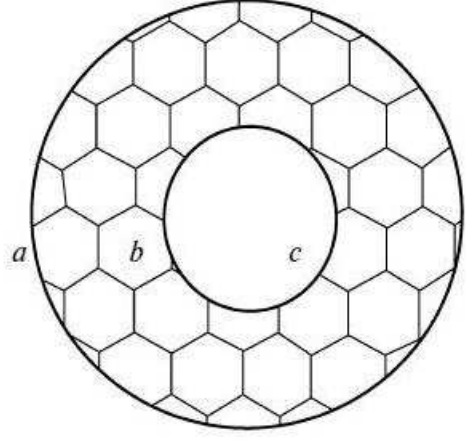


FIG. 2: The normal region II is divided by three areas a, b, c . The DBdG equations are solved for area b and determined by the "infinite mass" boundary conditions induced by $V(r) \rightarrow +\infty$ in a and c areas correspondently.

be extended and matched with superconducting regions. The two valleys $s \pm$ decouple, and we can solve equations separately for each valley, $H_s \psi_s = (\epsilon + sE_F) \psi_s$, $H_s = H_0 + sV(r)\sigma_z$. The term proportional to σ_z in Hamiltonian is a mass term confining the Dirac electrons in the area b . Rewrite the Hamiltonian in the cylindrical coordinates and since H_s commutes with $J_z = l_z + \frac{1}{2}\sigma_z$, its electron-eigenspinors ψ_e are eigenstates of J_z [35],

$$\Psi_e(r, \alpha) = \begin{pmatrix} \exp(id(n-1/2)\alpha) J_{d(n-1/2)}(k(\epsilon)r) \\ \exp(id(n+1/2)\alpha) J_{d(n+1/2)}(k(\epsilon)r) \end{pmatrix}$$

with eigenvalues n , where n is a half-odd integer, $n = d\frac{1}{2}, d\frac{3}{2}, \dots$ and $J_{d(n-1/2)}(k(\epsilon)r)$ is the Bessel function of $(n-1/2)$ order. In the $x-y$ plane d denotes moving direction of correspondent quasiparticle, $d = +$ for the quasiparticle moving toward $x = L/2$ and $d = -$ for the quasiparticle moving toward $x = -L/2$ direction correspondently. Further we are interested in to find zero energy states[22]. In this case the DBdG equations possess a general symmetry with respect to the change in the sign of energy[36],

$$\epsilon \rightarrow -\epsilon, \quad i\hat{\sigma}_y \hat{u}^* \rightarrow \hat{v}, \quad i\hat{\sigma}_y \hat{v}^* \rightarrow -\hat{u}, \quad (2)$$

where we denote $\hat{u} = (u_1, u_2)$ and $\hat{v} = (v_1, v_2)$. Thus, for a set of zero modes (\hat{u}_i, \hat{v}_i) enumerated by a certain index i we should have,

$$\hat{v}_i = i\hat{\sigma}_y \hat{u}_i^*, \quad \hat{u}_i = -i\hat{\sigma}_y \hat{v}_i^*. \quad (3)$$

In the same manner as for electron hole-spinors have view,

$$\Psi_h(r, \alpha) = \begin{pmatrix} -\exp(id(n+1/2)\alpha') J_{d(n+1/2)}(k'(\epsilon)r) \\ \exp(id(n-1/2)\alpha') J_{d(n-1/2)}(k'(\epsilon)r) \end{pmatrix}$$

with the definitions

$$\alpha(\epsilon) = \arcsin[\hbar v_F q / (\epsilon + E_F)], \quad (4)$$

$$\alpha'(\epsilon) = \arcsin[\hbar v_F q / (\epsilon - E_F)], \quad (5)$$

$$k(\epsilon) = (\hbar v_F)^{-1}(\epsilon + E_F) \cos(\alpha), \quad (6)$$

$$k'(\epsilon) = (\hbar v_F)^{-1}(\epsilon - E_F) \cos(\alpha), \quad (7)$$

The angle $\alpha \in (-\pi/2, \pi/2)$ is the angle of incidence of the electron (having longitudinal wave vector k), and α' is the reflection angle of the hole (having longitudinal wave vector k') [37, 38]. To obtain an analytical approximation of the spectrum, we use the asymptotic form of the Bessel functions for large r . This indeed is the desired limit as $rk(\epsilon) \approx r_{def}k(\epsilon) \propto r_{def}/L \ll 1$, where r_{def} is the defect radius (the radius of the area c) and determine for all eigenvalues $n = d\frac{1}{2}$. In this limit we impose the "infinite mass" boundary conditions at $y = 0, W$, for which $q_n = (n+1/2)\pi/W$ in the area b with $V(r) \rightarrow +\infty$ in the a and c areas consequently (see Fig. 3). Half-odd integer values n reflect the π Berry's phase of closed size of a single localized defect in graphene.

To obtain the subgap ($\epsilon < \Delta_0$) Andreev bound states, we now impose the boundary conditions at the graphene. The wave-functions in the superconducting and normal regions can be constructed as

$$\Psi_I = a_1\psi_I^+ + b_1\psi_I^-, \quad \Psi_{III} = a_2\psi_I^+ + b_2\psi_I^-, \quad (8)$$

$$\Psi_{II} = a\psi_{II}^{e+} + b\psi_{II}^{e-} + c\psi_{II}^{h+} + d\psi_{II}^{h-}. \quad (9)$$

where $a_1(b_1), a_2(b_2)$ are the amplitudes of right and left moving DBdG quasiparticles in region $I(III)$ and $a(b)$ and $c(d)$ are the amplitudes of right(left) moving electrons and holes, respectively, in the normal region[12]. These wave functions must satisfy the boundary conditions,

$$\Psi_I|_{x=-L/2} = \Psi_{II}|_{x=-L/2}, \quad \Psi_{II}|_{x=L/2} = \Psi_{III}|_{x=L/2} \quad (10)$$

Since the wave vector k_y parallel to the NS interface and different wave vectors in the y -direction are not coupled, we may solve the problem for a given $k_y = q$ and we can consider each transverse mode separately. To leading order in the small parameter $\Delta_0 L / \hbar v_F$ we may substitute $\alpha(\alpha') \rightarrow \alpha(0), k(\epsilon)(k'(\epsilon)) \rightarrow k(0)$. After some algebra we obtain equation

$$\cos(2\beta) \left\{ \frac{\sin^2(kL) - \cos^2(kL) \sin^2(\alpha)}{\cos^2(kL) \cos^2(\alpha) - 1} \right\} - \sin(2\beta) \left\{ \frac{\cos(kL) \sin(kL) \sin(\alpha)}{\cos^2(kL) \cos^2(\alpha) - 1} \right\} = \cos(\phi) \quad (11)$$

Eq. 11 differs from equation obtained for SNS junction without a single defect. Eliminating second term in Eq. 11 we could immediately yield reduction of the equation and it earns a essential form for weak SNS junctions[22]. The solution of Eq. 11 is a single bound state per mode,

$$\epsilon_n = \Delta_0 \sqrt{\frac{2}{A^2 + B^2} (-2CA^2 - B^2 + B\sqrt{1 - 4CA^2(C+1)})}$$

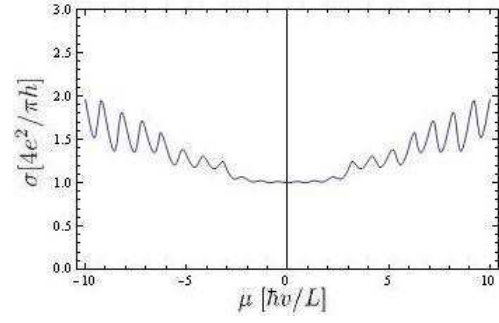


FIG. 3: Tunneling conductance (normalized per mode) of the graphene SNS junctions with a single localized defect versus Fermi energy, calculated from Eq. 14. The tunneling conductance exhibits oscillatory behaviour.

where

$$A = \frac{\sin^2(k_n L) - \cos^2(k_n L) \sin^2(\alpha)}{\cos^2(k_n L) \cos^2(\alpha) - 1}, \quad (12)$$

$$B = \frac{\cos(k_n L) \sin(k_n L) \sin(\alpha)}{\cos^2(k_n L) \cos^2(\alpha) - 1} \quad (13)$$

$$C = \frac{1}{2} + \frac{1}{D} \left(\frac{1}{2} - \sin^2\left(\frac{\phi}{2}\right) \right), \quad (14)$$

$$D = \frac{\cos^2(k_n L) \cos^2(\alpha) - 1}{\cos^2(k_n L) \cos^2(\alpha) - \cos(2k_n L)}, \quad (15)$$

We don't have a simple analytic expression for the ϕ -dependance but we obtained modified Andreev levels with the presence a single localized defect in the bulk. The conductance of the graphene strip is expressed through the transmission probability by the Landauer formula,

$$G = g_0 \sum_{n=0}^{n(\mu)} \tau_n, \quad g_0 = 4e^2/h, \quad (16)$$

where $n(\mu) \gg 1$ is given by $n(\mu) = \text{Int}(k_n W / \pi + 1/2)$. Substitution transmission probability into Eq. 16 gives the conductance (normalized per mode) versus Fermi energy (see in Fig. 3).

III. JOSEPHSON CURRENT

The Josephson current at zero temperature is given by

$$I(\phi) = -\frac{4e}{\hbar} \frac{d}{d\phi} \int_0^\infty d\varepsilon \sum_{n=0}^\infty \rho_n(\varepsilon, \phi) \varepsilon \quad (17)$$

where the factor of 4 accounts for the twofold spin and valley degeneracies. Substitution of $\rho_n(\varepsilon, \phi) = \delta[\varepsilon - \varepsilon_n(\phi)]$ into Eq. 17 gives the supercurrent due to the

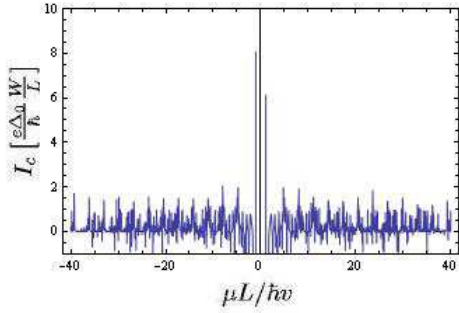


FIG. 4: Josephson current I_c of graphene-based SNS junction with single localized defect (length L short compared to the width W and superconducting coherence length ξ), as a function of the Fermi energy μ in the normal region.

discrete spectrum

$$I(\phi) = \frac{e\Delta_0}{h} \sum_{n=0}^{\infty} \frac{(3A_n^2 + B_n^2) \sin 2\phi}{8(A_n^2 + B_n^2)} \cdot \frac{1}{\sqrt{(3A_n^2 + B_n^2) \cos^2 \phi + (A_n^2 + B_n^2) B_n^2}} - \frac{A_n \sin \phi}{2(A_n^2 + B_n^2)} \quad (18)$$

Contributions to the supercurrent from the continuous spectrum are smaller by a factor L/ξ and may be neglected in the short-junction regime[6]. For $L \ll W$ the summation over n may be replaced by an integra-

tion. The resulting Josephson current I_c upon substitution $\phi \rightarrow \pi/2$ is plotted as a function of μ in Fig. 4.

IV. CONCLUSION

In conclusion, we have shown that a Josephson junction in graphene can carry a nonzero supercurrent even if the Fermi level is tuned to the point of zero carrier concentration. At this Dirac point, the current-phase relationship has a resonant behaviour due to single localized defect. This unusual "quasidiffusive" behaviour of the Josephson effect in undoped graphene should be observable in submicrometer scale junctions. It is found that the current has a peak-like structure, with alternating signs of the peaks. The resulting nonequilibrium Josephson current for a given phase difference thus oscillates as a function of chemical potential, i.e the junction displays so-called π -behaviour.

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